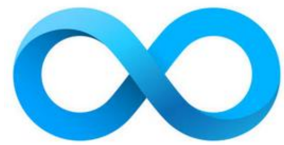
NUMBER THEORY



The set of **Natural Numbers** is the set

$$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$$

[The 3 dots mean 'go on forever and ever.'] Notice that there are infinitely many natural numbers. and although there is NO largest natural number, the smallest natural number is 1.

❑ *MULTIPLES*

If you were asked to "count by 7's, starting at 7, you would come up with the following set of numbers:

$$7, 14, 21, 28, 35, 42, \ldots$$

We call these numbers the **multiples of 7**. Notice that 7 divides into each of its multiples with a *remainder* of 0. (The 7 "goes in evenly.")

If you count by 12's, you get the **multiples of 12**:

$$12, 24, 36, 48, 60, \ldots$$

The **even numbers** can be defined as the **multiples of 2**:

$$2, 4, 6, 8, 10, 12, \ldots$$

What would you get if you consider all the **multiples of 1**? You arrive back at the **natural numbers**:

$$1, 2, 3, 4, 5, \ldots$$

If you'd like, you could add the number **0** to each list of multiples, since all the numbers 7, 12, 2, and 1 divide into 0 evenly (namely 0 times). But including 0 is not very useful, so we'll ignore 0 as a multiple.

Notes:

- ♦ Assuming that we ignore zero being a multiple, there is always a smallest multiple: the number itself. This implies that every multiple of a number is at least as big as the number itself.
- ♦ Every natural number has infinitely many multiples.

❑ COMMON MULTIPLES AND THE LCM

First Example: **Find the numbers that are multiples of both 3 and 5.**

That is, find the ***common multiples*** of 3 and 5.

Here are the multiples of **3**:

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45 . . .

And here are the multiples of **5**:

5, 10, 15, 20, 25, 30, 35, 40, 45, . . .

I'm now going to display the same two lists, the multiples of 3 and the multiples of 5 — but this time with boxes around the "common" multiples:

Multiples of **3**:

3, 6, 9, 12, $\boxed{15}$, 18, 21, 24, 27, $\boxed{30}$, 33, 36, 39, 42, $\boxed{45}$, . . .

Multiples of **5**:

5, 10, $\boxed{15}$, 20, 25, $\boxed{30}$, 35, 40, $\boxed{45}$, . . .

The *common multiples* are as follows:

15, 30, 45, . . . [These are the *common multiples* of 3 and 5.]

Notice that this list of common multiples, 15, 30, 45, . . ., has a *smallest* number, namely the **15**. Since 15 is a common multiple of 3 and 5, and because it's the *smallest* such common multiple, we say that

Sometimes the LCM of two numbers is simply their product.

15 is the ***Least Common Multiple* (*LCM*)** of 3 and 5.

We can abbreviate this statement by saying: $\boxed{\text{LCM}(3, 5) = 15}$

<u>Second Example:</u> **Find the LCM of 4 and 6.**

Multiples of **4**:

4, 8, $\boxed{12}$, 16, 20, $\boxed{24}$, 28, 32, $\boxed{36}$, . . .

Multiples of **6**:

6, $\boxed{12}$, 18, $\boxed{24}$, 30, $\boxed{36}$, . . .

The **common multiples** have been boxed, and the smallest of these is **12**. Therefore,

$\boxed{\text{LCM}(4, 6) = 12}$

Sometimes the LCM of two numbers is less than their product.

<u>Third Example:</u> **Find the LCM of 10 and 20.**

Multiples of **10**:

10, $\boxed{20}$, 30, $\boxed{40}$, 50, $\boxed{60}$, . . .

Multiples of **20**:

$\boxed{20}$, $\boxed{40}$, $\boxed{60}$, 80, . . .

I hope it's clear that 20 is the least common multiple:

Sometimes the LCM of two numbers is equal to the larger of them.

$$\boxed{\text{LCM}(10, 20) = 20}$$

<u>Fourth Example:</u> **Find the LCM of the three numbers: 6, 10, and 12.**

Multiples of **6**:

6, 12, 18, 24, 30, 36, 42, 48, 54, $\boxed{60}$, 66, 72 . . .

Multiples of **10**:

10, 20, 30, 40, 50, $\boxed{60}$, 70, 80, . . .

Multiples of **12**:

12, 24, 36, 48, $\boxed{60}$, 72, 84, . . .

The smallest common multiple we see is 60. So,

$$\boxed{\text{LCM}(6, 10, 12) = 60}$$

And what it means is this: **60** is the <u>smallest</u> number that 6, 10, and 12 divide into evenly.

Homework

GCF
PRIMES

Solutions